Using frozen hydrogen particles to observe rotating and quantized flows in liquid helium

A Dissertation
Presented to the Faculty of the Graduate School
of
Yale University
In Candidacy for the Degree of
Doctor of Philosophy

by
Gregory P. Bewley

Dissertation Director: Katepalli R. Sreenivasan

December 2006

Abstract

Using frozen hydrogen particles to observe rotating and quantized flows in liquid helium

Gregory P. Bewley
2006

We present a novel technique for tracing liquid helium flows, and use this technique to make observations of fluid dynamics in both the normal fluid and the superfluid phases of liquid helium. To visualize fluid motions, we create a suspension of frozen hydrogen particles with diameters on the order of one micron. We show theoretically that the hydrogen particles we generate can be used to make quantitative measurements of local flow velocities in turbulent liquid helium, and that these particles are the only ones we know of that are suitable for this purpose. In experimental work, we use the particles in normal liquid helium to examine the effect of the Coriolis force on the decay of classical turbulence using the Particle Image Velocimetry technique (PIV). We observe grid-generated turbulence in a steadily rotating frame and find that the evolution of the flow depends intimately on boundary conditions because of the production in the fluid of standing inertial wave modes of the container. Separately, we present what are very probably the first documented images of the cores of quantized vortices residing in the superfluid phase of liquid helium. Filaments we observe in the fluid are probably formed by the particle-trapping action of the quantized vortices. Although others have speculated how particles in superfluid helium could act as passive tracers of a flow, as they do in the normal fluid, our images indicate that the presence of particles in the superfluid may transform the topology of vortex tangles by stabilizing forks in the vortices.

Acknowledgements

I am grateful to my adviser, Katepalli Sreenivasan, for his steadfast support and guidance. I am also indebted to Dan Lathrop for accepting me in his lab and providing countless invaluable ideas. I wish to thank Rustom Bhiladvala for introducing me to science and lab work, and Chris White for initiating me in the techniques required to perform these experiments. My work would not have progressed without the engaging discussions I have had with Eric Kostelich, Narendra Kumar, Leo Maas, Joe Niemela, and Ladik Skrbek. I am especially beholden to Joe Vinen for the encouragement to experiment with superfluid helium, and to Russ Donnelly who proposed rotating the superfluid in order to confirm our observation of vortices. I also thank my committee members Ed Bolton, Marshall Long, Juan de la Mora, and Kailas Purushothaman. I greatly appreciate the assistance of Don Martin and my many inspiring lab mates both at Yale and at the University of Maryland. Finally, I would not have survived without the unreserved support of my friends and family. Thank you.

Contents

Acknowledgements **i**

Contents **ii**

List of figures **viii**

List of tables **xviii**

1. Introduction **1**

1.1 Scales of turbulent motion 2

1.2 Benefits of helium as a test fluid 3

1.2.1 Classical hydrodynamics 3

1.2.2 Quantum hydrodynamics 4

1.3 Available techniques for velocity measurement 5

1.4 The difficulties of observing liquid helium flows 6

1.4.1 Small length scales 6

1.4.2 Particle suspensions 7

1.5 Summary 8

2. Apparatus and methods **10**

2.1 The apparatus 10

2.1.1 The cryostat 12

2.1.2 The cooling protocol 12

2.1.3 The channel 13

2.1.4 The grid 14

2.1.5 The linear motor 15

2.1.6 Air bearing 15

2.1.7 Camera 16

2.1.8 Laser and optics 17

2.2 Particle image velocimetry 18
2.2.1 Brief description 18
2.2.2 Data processing 20
2.2.3 Limitation of the PIV technique at high Reynolds numbers 21
3. Particle selection **23**
3.1 Tracer particle fidelity 23
3.1.1 A method to evaluate a particle as a fluid tracer 23
3.1.2 Suggested alternate formulation 26
3.2 Large Froude and Stokes effects 28
3.2.1 Vibrated particles 28
3.2.2 Measurement of large scale flow properties 28
3.3 Particle clumping 30
3.3.1 Comparison of forces 31
3.3.2 Rate of coagulation 34
3.3.3 Suspension stabilization 35
3.3.4 Methods for dispersing particles 37
3.4 Evaluation of available seeding techniques 37
3.4.1 Commercially available particles – hollow glass spheres 37
3.4.2 Commercially available particles – solid beads 40
3.4.3 Commercially available particles – other types of particles 43
3.4.4 Review of previous work to make frozen particles 43
3.5 The first hydrogen injector 44
3.5.1 The basic properties of hydrogen 45
3.5.2 First attempts to make hydrogen particles 45
3.6 The improved hydrogen injector 48
3.6.1 The injector 48
3.6.2 Procedure 49
3.6.3 Results 50
3.7 Discussion 51
3.7.1 Particle volume fraction 51

3.7.2 Particle size 51

3.7.3 Model of injector function 53

3.7.4 Practical considerations 54

3.7.5 A note on the injection of particles into superfluid helium 55

3.7.6 Final analysis 55

3.8 Conclusions 57

4. Decaying grid turbulence 58

4.1 Background 58

4.1.1 Homogeneous and Isotropic Turbulence 58

4.1.2 Grid Turbulence 59

4.1.3 Theoretical predictions of the decay rate 60

4.1.4 Experimental observations 62

4.2 Methods 63

4.2.1 Protocol 63

4.2.2 Image timing 64

4.3 PIV measurements using hydrogen particles 65

4.3.1 PIV Data 66

4.3.2 Mean Flow 66

4.3.3 Turbulent energy decay 70

4.3.4 The strength of the mean flow 71

4.3.5 A comment on the magnitude of the observed variables 72

4.3.6 A note on thermal input 73

4.4 Conclusions 74

5. Turbulence in a rotating container 75

5.1 Rotating flows 75

5.1.1 The Coriolis acceleration and its effects 76

5.1.2 The parameter space investigated 78

5.1.3 Two-dimensional flows 78

5.1.4 Inertial waves 79

5.1.5 Inertial wave modes of a container 80

5.1.6 Reflecting inertial waves 81
5.1.7 Rotating, decaying homogeneous turbulence 82
5.2 Previous experimental work 82
5.3 Apparatus and methods 85
5.3.1 Experimental protocol for the cryogenic fluids 86
5.3.2 Spinup of the fluid 86
5.3.3 Liquid nitrogen 87
5.3.4 Water channel 88
5.4 Results 90
5.4.1 The energy decay 90
5.4.2 The mean flow 90
5.4.3 Transform of the mean flow 92
5.4.4 The circular apparatus 94
5.4.5 Revisiting the square channel data 96
5.4.6 The square channel with liquid helium 98
5.5 Discussion 100
5.5.1 The modes in liquid helium 100
5.5.2 The right experiment 100
5.5.3 The source of energy for the large scales 102
5.5.4 The parameter J 102
5.5.5 Why the modes have not been previously observed in this context 103
5.5.6 A note on the decay rate 104
5.6 Conclusions 104
6. Visualizing quantized vortex lines 105
6.1 Background 105
6.1.1 Basic physical properties of liquid helium 106
6.1.2 Quantized vortices 108
6.1.3 Quantization and the evolution of vortices 110
6.1.4 The two-fluid model 112
6.1.5 Residual vortices 114

6.1.6 A rotating superfluid 115
6.1.7 Flows in He II 116
6.1.8 Previous experimental observations 117
6.2 Methods 119
6.2.1 The formation of a superfluid 119
6.2.2 Preparation of the particle suspension in He II 120
6.2.3 Gathering data near the transition temperature 121
6.2.4 Gathering data in the rotating frame 122
6.3 Results 125
6.3.1 The filaments 125
6.3.2 Review of the assumptions 128
6.3.3 Evenly spaced particles 131
6.4 Discussion 133
6.4.1 A particle trapping mechanism 134
6.4.2 Trapping time and particle size 136
6.4.3 The driving force for the formation of vortex filaments 138
6.4.4 Limitations of the observation 139
6.4.5 The significance of branches 142
6.4.6 The dotted lines 143
6.5 Conclusions 144
7. Conclusions and future work **145**
7.1 Improvements to the apparatus 146
7.1.1 Heat input 146
7.1.2 Controlling experimental conditions 147
7.1.3 An additional window 148
7.1.4 Particle tracking 149
7.2 Particle characterization 149
7.3 Classical turbulence 150
7.3.1 Decaying turbulence in a non-rotating frame of reference 150
7.3.2 Rotating turbulence 150

7.4 Experiments in He II 151
7.4.1 Quenches through transition 152
7.4.2 Dotted lines 154
7.4.3 Tkachenko waves 154
7.4.4 Laser tweezers – plucking a quantized vortex 155
7.4.5 The wanderers 155
7.4.6 Branching and networks 155
7.4.7 Onset of turbulence 156
7.5 Closing remarks 158
Bibliography

List of figures

1.1: Viscosity and density for various fluids. All properties are given at atmospheric pressure. The properties of air are given at room temperature. For liquid nitrogen and liquid helium, they are given at their boiling points, 77 K and 4.2 K, respectively. 4

2.1: A picture of the apparatus showing the cryostat that rotates about the vertical axis on an air bearing, along with the camera and associated electronic equipment. Also shown are the linear motor that draws a grid through the channel inside the cryostat, and the optics used to illuminate particles suspended in the fluid. The cryostat is about one meter tall. 11

2.2: The drawing shows the interior of the cryostat. At the core is the 5 cm square channel through which we draw a grid. The main helium reservoir above it, as well as the annular liquid nitrogen jacket and outer wall of the cryostat each have circular cross sections. Between each concentric vessel is a common vacuum space. 13

2.3: An illustration of the grid shows its mesh spacing, M, and the grid stem by which it is pulled through the channel. 14

2.4: The line traces the path of the laser beam, the circles represent mirrors. All fixtures below the air bearing are fixed to an optical table; everything above the air bearing rotates together, including the beam when it is carefully aligned with the axis of rotation. 18

2.5: Examples of two images taken at successive times are shown at the top left. Small regions are taken from the images, called interrogation areas. The cross-correlation of the interrogation areas is computed, shown in the top right. The location of the peak gives the relative offset, Δx, of the particles between the two interrogation areas. A vector is assigned to the center of the interrogation area, whose magnitude and direction are given by $\Delta x/\Delta t$. This calculation is repeated for a chosen set of pairs of interrogation areas in the two images. 19

3.1: The van der Waals potential as a function of sphere separation is the dashed line. We use Hamaker's constant for polystyrene, $A = 8 \times 10^{-20}$ J, but the Hamaker constant for most materials is of the same order. An exception is liquid helium, and since Hamaker's constant for liquid helium is much smaller (Paalanen and Iye 1985), we do not need to account for it as the intervening fluid (Heimenz and Rajagoplan 1997). 31

3.2: The ratio of the attractive van der Waals force to the fluid shear induced separation force for polystyrene spheres in a turbulent flow at $Re_\lambda = 400$. The ratio is given as a function of the sphere's radius for several surface separations. The surface separation is expressed as a fraction of the sphere radius, imagining that some surface roughness keeps the spheres further apart than if they were perfect spheres. For particles smaller than a few microns, the van der Waals force is much larger than our estimated upper bound for the separation force. 34

3.3: An image of a suspension of hollow glass spheres in liquid helium. 38

3.4: The chart shows the Stokes and Froude numbers as they evolve in a turbulent flow generated by a grid with 7.2 mm mesh spacing, M, drawn at $U = 2.0$ m/s in liquid helium. The parameters are calculated for a 9 µm hollow glass sphere. The horizontal line is the maximum value for either parameter determined in section 3.2, and that either one parameter or the other is larger at any moment indicates that the particle does not trace fluid motions accurately. 40

3.5: On the left are clumps of 2 μm polystyrene particles in liquid helium. On the right are identical particles in water, using a surfactant to disperse them. 41

3.6: The chart shows the Stokes and Froude numbers as they evolve in a turbulent flow generated by a grid with 7.2 mm mesh spacing, M, drawn at $U = 2.0$ m/s in liquid helium. The parameters are calculated for a clump of polystyrene particles with a characteristic diameter of 2.4 μm. The horizontal line is the maximum value for either parameter determined in section 3.2, and that either one parameter or both is larger at any moment indicates that the particle clump does not trace fluid motions accurately. 42

3.7: The liquid hydrogen injector consists of concentric stainless steel tubes that are kept from touching each other with Teflon spacers every 6 cm. A vacuum develops between them when the injector is cooled. Hydrogen gas is introduced at the top. A wire along the length of the core and a wire wrapped around the nozzle can separately be heated to keep the interior of the injector above the freezing point of hydrogen. 46

3.8: On the left is a liquid hydrogen jet in liquid helium. On the right are the resultant solid hydrogen "snow flakes." 47

3.9: The schematic shows the design of our diluted hydrogen injector. Hydrogen gas diluted with helium is introduced at the top, and enters the cryostat through the inner of two concentric stainless steel tubes. The mixture cools on passage through the injector and exits below the free surface of liquid helium. 49

3.10: Hydrogen particles generated by diluting hydrogen gas with helium in a 1 : 10 ratio are shown. The particle size distribution appears polydisperse, but overall the particles appear smaller and more numerous than those shown in figures 3.3, 3.5, and 3.8. 50

3.11: The chart shows the Stokes and Froude numbers as they evolve in a turbulent flow generated by a grid with 7.2 mm mesh spacing, M, drawn at

U = 2.0 m/s in liquid helium. The parameters are calculated for hydrogen particles with a characteristic diameter of 3 μm. The horizontal line is the maximum value for either parameter determined in section 3.2; note that the scale in this chart is different than in the comparable figures 3.3 and 3.6. In this example, the hydrogen particles are suitable for making quantitative measurements of local fluid velocities in the turbulent flow. 52

4.1: Images are taken at each tick mark. Each set of five ticks is called a packet, and the image frames are equispaced in time within each packet. We program the time between frames within each packet, and the time between packets, to grow larger as the turbulent velocities decay, such that the particles move a constant distance between images in a packet. 64

4.2: PIV vectors computed from two images taken 1 ms apart about 140 ms after the grid has passed the field of view at 1.0 m/s. Processing on images of hydrogen particles is done according to the description in section 2.2, with an interrogation area size of 24 × 24 pixels in the final pass. 65

4.3: A cross section of the channel, and a sketch of the large scale flow the grid generates as it is towed along the length of the channel. In the area we observe through the window, the flow appears uniform, though it must recirculate down the sides of the channel. 67

4.4: A sketch of one of four flush-fitting windows in the side walls of the channel. The fine line traces the outline of the original flat windows. The grid passes on the right in this drawing, and the vacuum space is on the left. The vacuum seal is formed by crushing an indium wire between the window and the channel wall. 68

4.5: The mean flow as a fraction of the grid velocity, U_g, using liquid nitrogen in the channel, and using different grid designs. The solid lines are the for the grid in its original configuration, drawn from its center by a rod, the dashed lines are for the grid drawn from its corners, and the dotted lines are for the original configuration, but with flush fitting windows. The strength of the currents is comparable despite the modifications. 69

4.6: The streamwise mean flow with a 0.33 mm wire around the perimeter of the grid, in both liquid nitrogen and helium. The transverse mean flow remains largely unchanged, is always smaller than 1% of the grid velocity, and is not shown. 70

4.7: The decay of kinetic energy per unit mass in the flow generated by a grid passing through liquid helium as a function of time since the grid has passed. Variables are normalized by the grid parameters, the grid velocity, U_g = 1.0 m/s, and the mesh spacing, M = 7.2 mm. 71

4.8: The fluctuating energy relative to the energy in the mean for the same flow as in figure 4.7. The streamwise fluctuations are compared to the streamwise mean, and the transverse fluctuations are compared to the transverse mean. 72

5.1: The Reynolds-Rossby number parameter plane using the large scales of the flow, such that $Ro = u/\Omega L$, and $Re = uL/\nu$, showing the limits in each parameter we can achieve simultaneously using our apparatus. Here, the variable u is the root-mean-square flow velocity, and L is the characteristic size of the energy-containing scales, which (for convenience) we take to be the mesh spacing of the grid, M. The bold lines are trajectories taken from data presented in this chapter, but any combination of larger Rossby number and smaller Reynolds number can be achieved by choosing appropriate initial conditions. 77

5.2: The circular cylindrical channel is put in place of the cryostat and is filled with water. The position of the false ceiling is adjustable. The indicated window faces the camera, while the laser passes through the curved wall of the channel. 88

5.3: The crosses are the decay of kinetic energy in a channel rotating at about 1 Hz filled with liquid nitrogen, after being agitated with a grid. The solid line is from data acquired in the same way, but without system rotation, at a mesh Reynolds number of 72,000. In the stationary case, the decay of kinetic energy is slightly steeper than t^{-1}, which is expected of a lower Reynolds number flow, as described in chapter 4. The decay with rotation shows strong fluctuations in time. 91

5.4: The mean flow for the same data as in figure 5.3. Streamwise flows are in the direction of grid motion. 91

5.5: The mean flow shown in figure 5.4 relative to the stochastic fluctuations in the local velocity. The solid line is the streamwise velocity component; the dashed line is the transverse component. The oscillations are of comparable magnitude to the fluctuations throughout the decay. 92

5.6: The transform of the repeatable mean flow shown in figure 5.4, generated by a grid drawn through liquid nitrogen in a square channel rotating at $\Omega = 0.98$ Hz. The energy in each transform lies predominantly below a frequency that is twice the system rotation rate, as must be the case for inertial waves. 93

5.7: The mean flow in the water channel for $h = 80$ cm, comparable to figure 5.4 of the same measurement, but using liquid nitrogen in a square channel. 94

5.8: The transform of the streamwise, or axial, mean flow shown in figure 5.7. The vertical lines are the locations of inertial wave modes predicted by Kelvin's linear, inviscid theory given in section 5.1.4. 95

5.9: We plot as circles the frequencies of the dominant peaks in spectra, such as the one in figure 5.8, for a series of aspect ratios, h/b, where h is the length of the channel along the axis of rotation, and b is its radius. The curves are calculated according to (5.1.8) for different combinations of axial and transverse wavenumbers. 96

5.10: The frequency spectrum of velocity fluctuations in a rotating square channel filled with liquid nitrogen. The vertical lines are predictions made by Maas (2005), whose [axial, transverse] wavenumbers are given in the order of increasing frequency. Most of the peaks correspond to successively higher axial wavenumbers. 97

5.11: The decay of energy in liquid helium with and without rotation. The solid curve is from figure 4.6, for a mesh Reynolds number of 283,000. The crosses trace a curve similar to that observed in liquid nitrogen in figure 5.3. 98

5.12: The mean flow in liquid helium, relative to the fluctuations. The solid line is the streamwise velocity component; the dashed line is the transverse component. The oscillations are of comparable magnitude to the fluctuations throughout the decay. 99

5.13: The solid curve is the transform of the streamwise mean flow in liquid helium. It shows well defined peaks, all lying below twice the rotation rate. We show the data from figure 5.6 using liquid nitrogen as a dashed curve. The peaks of the two curves coincide at high frequencies, but not below $\omega/2\Omega \approx 0.4$. We indicate the frequencies of some high axial wavenumber inertial wave modes as suggestions, but there are many other modes with frequencies that do not lie near peaks. 99

6.1: Phase diagram for helium near the lambda point, from Donnelly (1967). 107

6.2: The specific heat of liquid helium as a function of temperature at the saturated vapor pressure, from data collected by Donnelly and Barenghi (1998). 107

6.3: The two curves shown are the densities of the normal fluid and superfluid in the two fluid model of He II, at its saturated vapor pressure. The densities sum to the whole fluid's density, which is almost constant as a function of temperature and nearly equals 0.14 g/cc. At the lambda point, He II is entirely accounted for by the normal fluid part. At absolute zero, He II is a pure superfluid. 114

6.4: Data taken from Yarmchuk et al. (1979) is shown. The spots are the intersections of quantized vortices in a steadily rotating bucket where they intersect with the free surface. The rotation rate of the bucket generally increases from the left to the right in the sequence of images, as they are oriented here. 118

6.5: The time line of an experiment is portrayed in the figure. The relative times, temperatures, and the slopes of the curve are consistent with a typical experiment, though the temperature can be allowed to fall significantly further by leaving open the vacuum pumping valve for longer than indicated. 120

6.6: The times and temperatures for images taken during four selected, but typical, trials. The curves can be compared to the one in figure 6.5. We estimate the net cooling rate as 350 mW from the slope of the curves and the volume of liquid helium. Using this cooling rate, we integrate the heat capacity in figure 6.2 to produce the solid curve shown. The offset of this curve from the data can be attributed to systematic error in the thermometer. However, the origin of the differences of shape of the curves is unknown, and may be due to overturning of the fluid as its density changes near the lambda point. 122

6.7: Panel (a) shows a suspension of hydrogen particles just above the transition temperature. Panels (b), (c), and (d) show similar hydrogen particles after the fluid is cooled below the lambda point. Some particles have collected along branching filaments, while other are randomly distributed as before. Fewer free particles are apparent in (b), (c), and (d) only because the light intensity is reduced to highlight the brighter filaments in the image. The nature of the branching filaments is discussed in section 6.4. 124

6.8: The picture shows an example of particles arranged along vertical lines. The system rotation axis is vertical. On the left is the original, and on the right is an enhanced and inverted version. The spacing of lines is remarkably uniform, although there are occasional distortions of the lattice and possible points of intersection. 126

6.9: The axis of rotation is normal to the page. We assume a triangular array of vortices; their intersections with the horizontal plane are indicated by dots. The light sheet also intersects the page, and its cross section is represented by the dashed lines at the orientation relative to the lattice assumed in our calculation of line density. 127

6.10: We compute the density of lines made visible by particles as in figure 6.8, according (6.3.1) and the assumptions discussed in the text. The error bars are the standard deviation of all the measurements made at each rotation rate. Feynman's rule is given by (6.1.14), and the significance of the dashed line is described in section 6.3.2. 128

6.11: As in figure 6.9, we show the intersection of vortices with the horizontal plane as spots. However, consider a light sheet orientation different from the simplest one that illuminates a different line spacing than the minimum lattice spacing. 130

6.12: A series of images of particles regularly spaced along lines, taken during three independent excursions below the phase transition temperature by steadily cooling the fluid. In (a), the temperature is 32 mK below the lambda point, and about 350 s have passed since attaining the transition temperature. In (b), the temperature is 25 mK below the lambda point, and about 300 s have elapsed. In (c), the temperature is 28 mK below the lambda point, and about 400 s have elapsed. Temperatures are accurate to within 5 mK, and times within 40 s. Dotted lines such as these were not observed before or after the stated times, or at other temperatures. 132

6.13: The circles show the mean of histograms of particle spacings along dotted lines, where the histogram for each line is normalized by the mean spacing along that line. The solid line is the distribution of mean spacings among the different dotted lines normalized by the overall mean spacing. The spacing along a particular line is seen to have a narrower distribution than the difference in spacings between different lines. 133

6.14: The chart shows the ratio of the total drag on a vortex due to mutual friction to the drag caused by the presence of particles trapped on the line. We include a factor σ, describing the spacing of the particles. For σ = 1, the particles are next to each other, for σ = 10, they are 10 diameters apart. Particle drag is always stronger than mutual friction if the particles entirely cover the line. 141

7.1: The panels show the coarsening of a hydrogen particle suspension in liquid helium as the fluid is cooled through the superfluid phase transition. In (a), the temperature is 2.173 K, in (b) it is 2.170 K, in (c) it is 2.167 K, and in (d) it is 2.163 K. 153

7.2: These images were taken while superfluid liquid helium had reached a state of steady rotation about the vertical axis, under similar conditions as figure 6.8. It is likely, however, that the presence of a higher concentration of particles has caused the lattice of quantized vortices to pinch together in places. 156