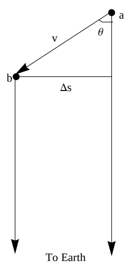
PHYS 411

Discussion Class 13

07 December 2007

1. Consider a star traveling with speed v at an angle θ relative to our line of sight (see the figure). What is its *apparent speed* across the sky? What angle θ_0 gives the *maximum* apparent speed? Show that the apparent speed can be much greater than the speed of light, even though v itself is always less than c.



[**Hint**: Suppose the light signal from point b reaches us at a time Δt after the signal from a, and the star has meanwhile advanced a distance Δs across the sky; then $u = \Delta s/\Delta t$ is the apparent speed. Show that as $v \to c,\ u \to \infty$.]

2. Let us consider a certain inertial frame $\mathcal{S}$ in which the electric field $\mathbf{E}$ and the magnetic field $\mathbf{B}$ are neither parallel nor perpendicular, at a particular space-time point. Show that in a different inertial system $\overline{\mathcal{S}}$, moving relative to $\mathcal{S}$ with velocity $\mathbf{v}$ given by

$$\frac{\mathbf{v}}{1+v^2/c^2} = \frac{\mathbf{E}\times\mathbf{B}}{B^2+E^2/c^2},$$

the fields $\overline{\mathbf{E}}$ and $\overline{\mathbf{B}}$ are *parallel* at that point. Can there be any frame in which the fields are *perpendicular*?

[**Hint**: For simplicity, choose your axes so that $\mathbf{E} = (0, 0, E)$; $\mathbf{B} = (0, B\cos\phi, B\sin\phi)$. Use Eq. (12.108) to calculate $\overline{\mathbf{E}}$ and $\overline{\mathbf{B}}$ in the frame $\overline{\mathcal{S}}$ moving at speed v in the x-direction.]

Solution

1.

$$\begin{cases} \text{light signal leaves } a \text{ at time } t'_a; & \text{arrives at earth at time } t_a = t'_a + d_a/c, \\ \text{light signal leaves } b \text{ at time } t'_b; & \text{arrives at earth at time } t_b = t'_b + d_b/c \end{cases}$$

Here d_a is the distance from a to earth, and d_b from b to earth. So

$$\begin{aligned} \Delta t &= t_b - t_a = t'_b - t'_a + \frac{d_b - d_a}{c} = \Delta t' + \frac{(-v\Delta t' \cos\theta)}{c} = \Delta t' \left(1 - \frac{v}{c}\cos\theta\right) \\ \Delta s &= v\Delta t' \sin\theta = \frac{v \sin\theta \Delta t}{1 - \frac{v}{c}\cos\theta} \\ \Rightarrow\ u &= \frac{\Delta s}{\Delta t} = \frac{v\sin\theta}{1 - \frac{v}{c}\cos\theta} \end{aligned}$$

To find the angle at which u is maximum,

$$\begin{aligned} 0 &= \frac{du}{d\theta} = \frac{v\left[\left(1 - \frac{v}{c}\cos\theta\right)(\cos\theta) - \sin\theta\left(\frac{v}{c}\sin\theta\right)\right]}{\left(1 - \frac{v}{c}\cos\theta\right)^2} \\ \Rightarrow & \quad \left(1 - \frac{v}{c}\cos\theta\right)\cos\theta = \frac{v}{c}\sin^2\theta \\ \Rightarrow & \quad \cos\theta = \frac{v}{c}, \quad \text{or} \\ & \quad \theta_0 = \cos^{-1}(v/c) \end{aligned}$$

At this angle,

$$u_0 = \frac{v\sqrt{1 - v^2/c^2}}{1 - v^2/c^2} = \frac{v}{\sqrt{1 - v^2/c^2}}$$

As $v \to c,\ u \to \infty$, even though $v < c$.

2. We choose the axes such that $\mathbf{E}$ points in the z direction and $\mathbf{B}$ in the yz plane:

$$\mathbf{E} = (0, 0, E);\ \mathbf{B} = (0, B\cos\phi, B\sin\phi).$$

Using Eq. (12.108), the transfomred fields in the $\overline{\mathcal{S}}$ frame moving at speed v in the x direction:

$$\overline{\mathbf{E}} = (0, -\gamma v B\sin\phi, \gamma(E + vB\cos\phi));\ \overline{\mathbf{B}} = (0, \gamma(B\cos\phi + \frac{v}{c^2}E), \gamma B\sin\phi).$$

$\overline{\mathbf{E}} \parallel \overline{\mathbf{B}}$ provided

$$\begin{aligned} & \frac{E_y}{B_y} = \frac{E_z}{B_z}, \\ \text{or} \quad & \frac{-\gamma v B\sin\phi}{\gamma(B\cos\phi + \frac{v}{c^2}E)} = \frac{\gamma(E + vB\cos\phi)}{\gamma B\sin\phi}, \\ \text{or} \quad & -vB^2\sin^2\phi = (B\cos\phi + \frac{v}{c^2}E)(E + vB\cos\phi), \\ \text{or} \quad & \frac{v}{1 + \frac{v^2}{c^2}} = -\frac{EB\cos\phi}{B^2 + E^2/c^2} \end{aligned}$$

Now $\mathbf{E} \times \mathbf{B} = -EB \cos\phi \hat{\mathbf{x}}$. So

$$\frac{\mathbf{v}}{1 + v^2/c^2} = \frac{\mathbf{E} \times \mathbf{B}}{B^2 + E^2/c^2}$$

There can be *no* frame in which $\mathbf{E} \perp \mathbf{B}$, for $(\mathbf{E} \cdot \mathbf{B})$ is invariant, and since it is not zero in $\mathcal{S}$ it can't be zero in $\overline{S}$.