1. Calculate the electric field outside of a solid sphere, radius $R$, with a charge distribution $\rho = \rho_0$ (constant) inside. What is the dominant multipole moment for this charge distribution?

\textbf{Ans:} Using Gauss’s law over a sphere of radius $r > R$,

$$\oint E \cdot da = \frac{Q_{\text{enc}}}{\epsilon_0}$$

$$\Rightarrow 4\pi r^2 E_r = \frac{\frac{4}{3} \pi R^3 \rho_0}{\epsilon_0}$$

$$\Rightarrow E_r = \frac{R^3 \rho_0}{3 \epsilon_0 r^2}$$

The dominant term is the \textit{monopole} term as $Q_{\text{total}} \neq 0$.

2. What is the electric field from a charged line on the $z$ axis with a charge per unit length $\lambda$? What is the force due to that electric field on a charge $q$ at $s = a$, $\phi = 0$, and $z = 0$?

\textbf{Ans:} Use Gauss’s law over a cylindrical surface with length $L$ and radius $s$:

$$\oint E \cdot da = \frac{Q_{\text{enc}}}{\epsilon_0}$$

$$\Rightarrow 2\pi s L E_s = \frac{\lambda L}{\epsilon_0}$$

$$\Rightarrow E_s = \frac{\lambda}{2\pi \epsilon_0 s}$$

The force on charge $q$ at $s = a$ is given by

$$\mathbf{F} = q \mathbf{E} = \frac{q \lambda}{2\pi \epsilon_0 a} \mathbf{s}$$

3. What is the electric potential exterior to a sphere, radius $R$, with the potential $\Phi_0 = 3 \cos \theta + 2$ on its surface?
Ans: The general solution for the potential is given by

\[ \Phi(r, \theta) = \sum_{l=0}^{\infty} \left( A_l r^l + \frac{B_l}{r^{l+1}} \right) P_l(\cos \theta) \]

For exterior of the sphere, \( A_l = 0 \ \forall \ l \). At the surface, we have \( \Phi_0 = 3P_1 + 2P_0 \). Then continuity of \( \phi \) implies that \( B_0 = 2R \), \( B_1 = 3R^2 \) and all other \( B_l = 0 \). Thus the potential exterior to the sphere is

\[ \Phi(r, \theta) = \frac{2R}{r} + \frac{3R^2}{r^2} \cos \theta \quad (r > R) \]

4. Calculate the capacitance between two concentric cylinders of radius \( a \) and \( b \) which are of length \( L \). Assume \( L \gg a \) and \( L \gg b \) such that end effects are not important.

Ans: As already seen from Problem 2,

\[ E_s = \frac{\lambda}{2\pi \varepsilon_0 s} = \frac{Q}{2\pi \varepsilon_0 s L} \]

Then the potential difference between the two cylinders is given by

\[ V = \int_a^b E_s ds = \frac{Q}{2\pi \varepsilon_0 L} \int_a^b \frac{ds}{s} = \frac{Q \ln(b/a)}{2\pi \varepsilon_0 L} \]

So the capacitance is given by

\[ C = \frac{Q}{V} = \frac{2\pi \varepsilon_0 L}{\ln(b/a)} \]

5. If a point charge \( Q \) is placed at the center of a grounded conducting spherical shell, radius \( R \). What is the electrical potential everywhere?

Ans: The potential is given by

\[ \Phi = \begin{cases} \frac{Q}{4\pi \varepsilon_0 r} + \Phi_0, & r < R \\ 0, & r > R \end{cases} \]

where \( \Phi_0 \) is a constant. Continuity at the surface implies

\[ \Phi_0 = -\frac{Q}{4\pi \varepsilon_0 R} \]

So the potential everywhere is given by

\[ \Phi = \begin{cases} \frac{Q}{4\pi \varepsilon_0} \left( \frac{1}{r} - \frac{1}{R} \right), & r \leq R \\ 0, & r \geq R \end{cases} \]