PHYS 411

Discussion Class 6

(MIDTERM QUESTIONS)

12 October 2007

1. Calculate the electric field outside of a solid sphere, radius R, with a charge distribution $\rho = \rho_0$ (constant) inside. What is the dominant multipole moment for this charge distribution?

   Ans: Using Gauss's law over a sphere of radius $r > R$,

$$\begin{aligned} \oint \mathbf{E} \cdot d\mathbf{a} &= \frac{Q_{\text{enc}}}{\epsilon_0} \\ \Rightarrow \quad 4\pi r^2 E_r &= \frac{\frac{4}{3}\pi R^3 \rho_0}{\epsilon_0} \\ \Rightarrow \quad E_r &= \frac{R^3 \rho_0}{3\epsilon_0 r^2} \end{aligned}$$

   The dominant term is the *monopole* term as $Q_{\text{total}} \neq 0$.

2. What is the electric field from a charged line on the z axis with a charge per unit length λ? What is the force due to that electric field on a charge q at $s = a$, $\phi = 0$, and $z = 0$?

   Ans: Use Gauss's law over a cylindrical surface with length L and radius s:

$$\begin{aligned} \oint \mathbf{E} \cdot d\mathbf{a} &= \frac{Q_{\text{enc}}}{\epsilon_0} \\ \Rightarrow \quad 2\pi s L E_s &= \frac{\lambda L}{\epsilon_0} \\ \Rightarrow \quad E_s &= \frac{\lambda}{2\pi\epsilon_0 s} \end{aligned}$$

   The force on charge q at $s = a$ is given by

$$\mathbf{F} = q\mathbf{E} = \frac{q\lambda}{2\pi\epsilon_0 a}\hat{\mathbf{s}}$$

3. What is the electric potential exterior to a sphere, radius R, with the potential $\Phi_0 = 3\cos\theta + 2$ on its surface?

Ans: The general solution for the potential is given by

$$\Phi(r,\theta) = \sum_{l=0}^{\infty} \left(A_l r^l + \frac{B_l}{r^{l+1}} \right) P_l(\cos\theta)$$

For exterior of the sphere, $A_l = 0 \;\forall\; l$. At the surface, we have $\Phi_0 = 3P_1 + 2P_0$. Then continuity of ϕ implies that $B_0 = 2R$, $B_1 = 3R^2$ and all other $B_l = 0$. Thus the potential exterior to the sphere is

$$\Phi(r,\theta) = \frac{2R}{r} + \frac{3R^2}{r^2}\cos\theta \quad (r > R)$$

4. Calculate the capacitance between two concentric cylinders of radius a and b which are of length L. Assume $L \gg a$ and $L \gg b$ such that end effects are not important.

Ans: As already seen from Problem 2,

$$E_s = \frac{\lambda}{2\pi\epsilon_0 s} = \frac{Q}{2\pi\epsilon_0 s L}$$

Then the potential differnece between the two cylinders is given by

$$V = \int_a^b E_s ds = \frac{Q}{2\pi\epsilon_0 L}\int_a^b \frac{ds}{s} = \frac{Q\ln(b/a)}{2\pi\epsilon_0 L}$$

So the capacitance is given by

$$C = \frac{Q}{V} = \frac{2\pi\epsilon_0 L}{ln(b/a)}$$

5. If a point charge Q is placed at the center of a grounded conducting spherical shell, radius R. What is the electrical potential everywhere?

Ans: The potential is given by

$$\Phi = \begin{cases} \frac{Q}{4\pi\epsilon_0 r} + \Phi_0, & r < R \\ 0, & r > R \end{cases},$$

where Φ_0 is a constant. Continuity at the surface implies

$$\Phi_0 = -\frac{Q}{4\pi\epsilon_0 R}$$

So the potential everywhere is given by

$$\Phi = \begin{cases} \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r} - \frac{1}{R}\right), & r \leq R \\ 0, & r \geq R \end{cases}$$