Magnetohydrodynamics in experimental S_1T_1 flow

Douglas H. Kelley, Daniel S. Zimmerman, Santiago Andrés Triana, Andreas Tilgner*, and Daniel P. Lathrop
Departments of Physics and Geology, IREAP, IPST, University of Maryland | * Institute of Geophysics, University of Göttingen

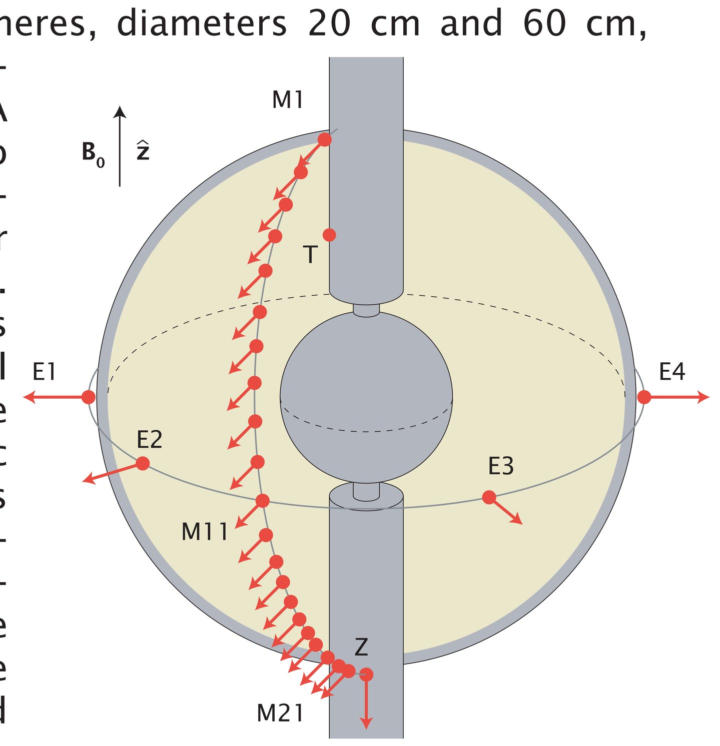
Abstract

The physics of rotating, electrically conductive fluids allows for a vast array of dynamical behaviors, many of which are relevant to geophysical flows in the Earth's core: inertial waves, Alfvén waves, magnetocoriolis (MC) waves, dynamo action, and geomagnetic reversals, among others. We report experimental observations of various magnetohydrodynamic behaviors, including waves, in spherical geometry. One set of results (Kelley *et al.*, 2007) shows inertial waves in Earth-like spherical Couette geometry. Evidence suggests that the motions are forced by over-reflection at a shear layer in the fluid. A second set of results, taken in a spherical shell with forcing by a hydrofoil propeller, shows more complex wave-like behavior. Multiple frequencies emerge both temporally and spatially, and initial analysis suggests that these motions are a superposition of MC waves.

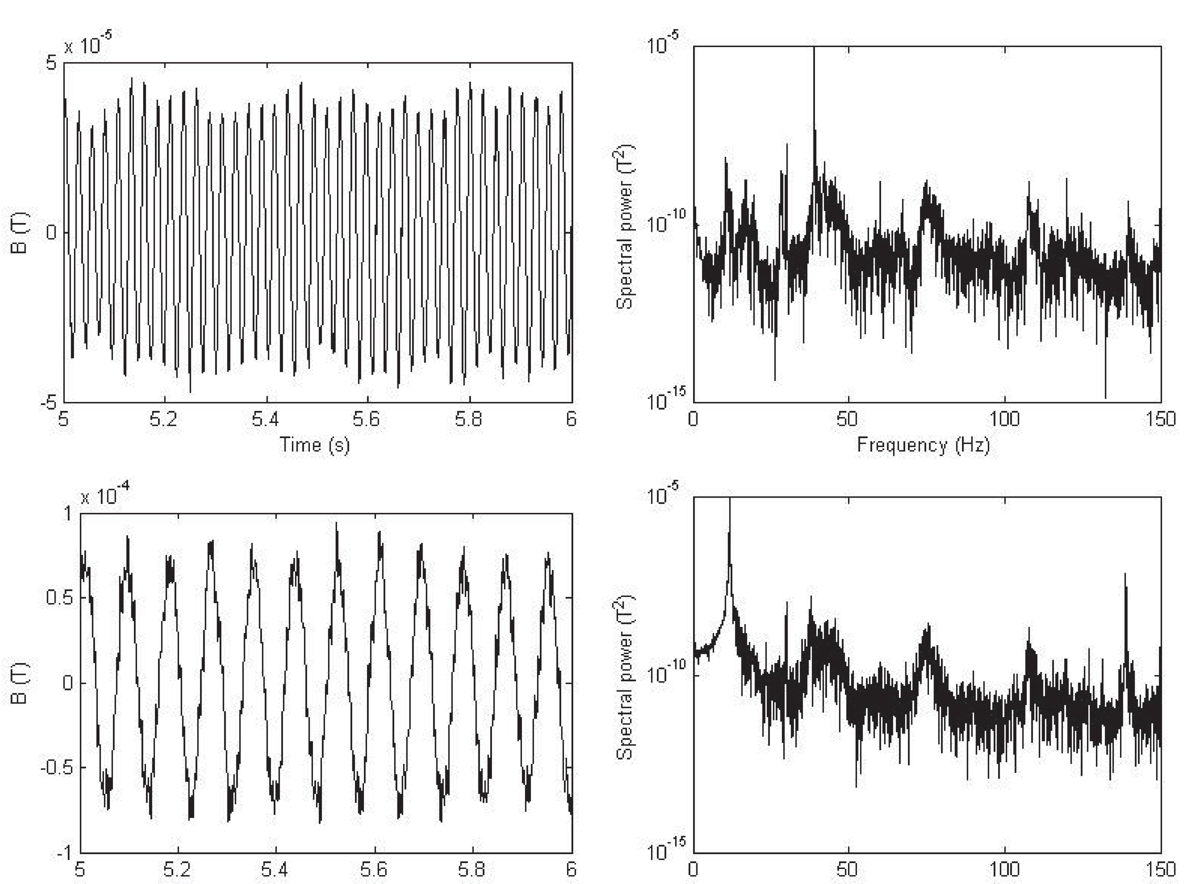
Past work

Experimental setup

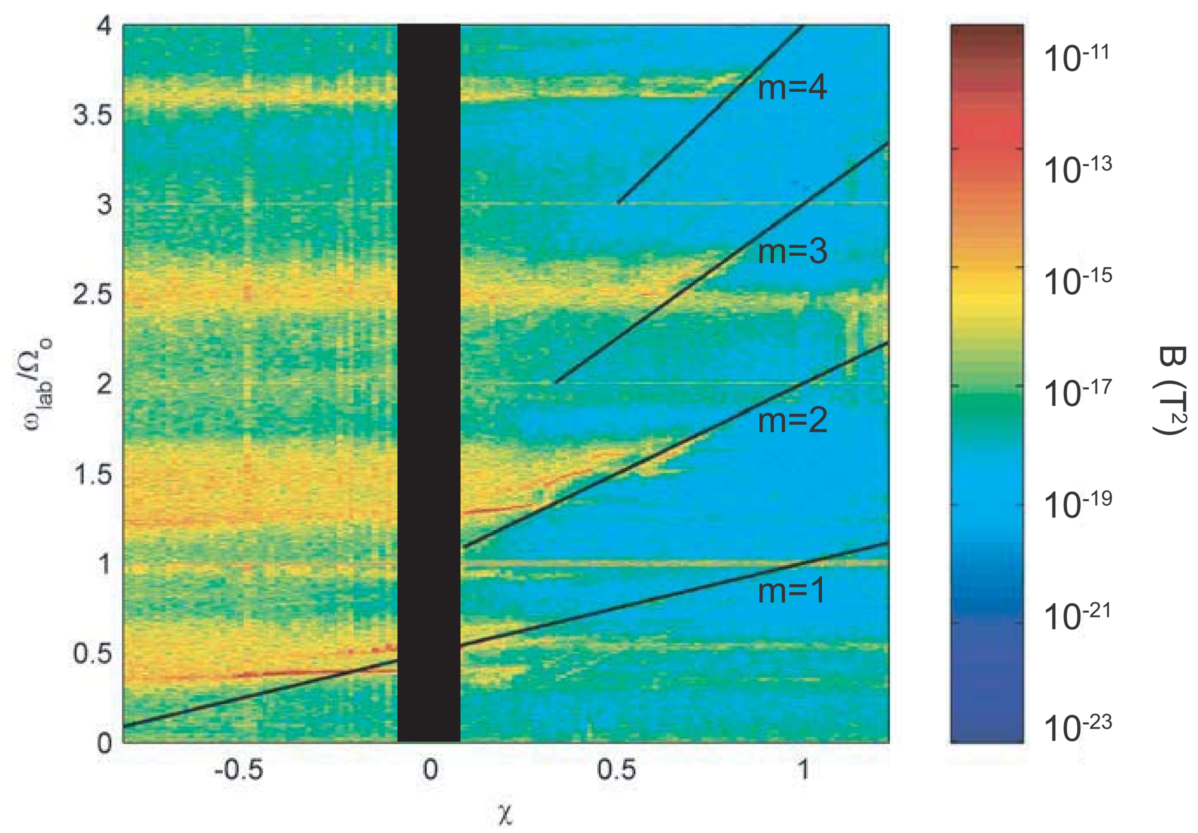
Our experimental work in spherical Couette flow is directly motivated by the geometry of Earth's core. The apparatus, shown here, is designed to mimic the geometry of the core and has two independently rotating concentric spheres, diameters 20 cm and 60 cm, with 110 L of liquid sodium filling the gap between them. A DC magnetic field $\mathbf{B}_0$ parallel to the rotation axis, with magnitude 150 G, is applied by a pair of external electromagnets. The bulk of our data comes from an array of 25 Hall probes, which measure the component of the magnetic field along a cylindrical radius (B_s) at 21 locations along a meridian and four additional locations around the equator. One more Hall probe measures the axial component of the field (B_z) near the lower pole (see diagram).



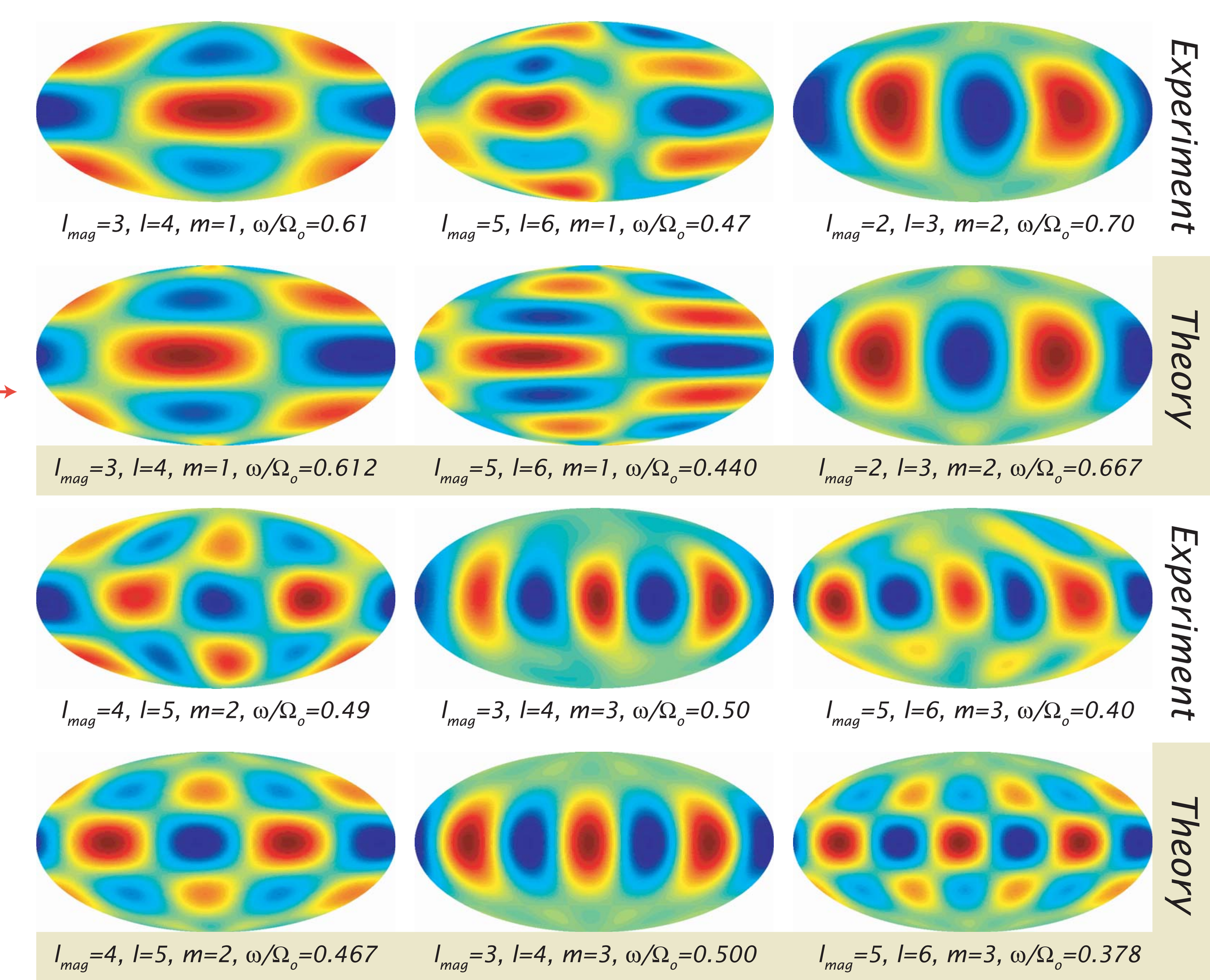
Results



Two time series from a single Hall probe, along with their power spectra. Experimental parameters are $(\Omega_i,\Omega_o)/2\pi=(5.7,29.9)$ Hz, upper; $(\Omega_i,\Omega_o)/2\pi=(-12.2,29.9)$ Hz, lower.



Spectrogram of inner sphere ramp showing oscillatory modes. Normalized rotation rate $\chi=\Omega_i/\Omega_o$ varies along the horizontal axis, and normalized signal frequency (from an equatorial Hall probe) ω_{lab}/Ω_o varies along the vertical. Color indicates power spectral density. Black lines show Mach 2 boundaries; see below. Here $\Omega_o/2\pi=29.9$ Hz.



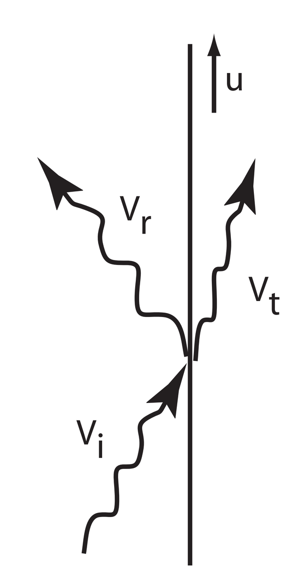
Induction B_s/B_0 at the surface, over one revolution of the dominant pattern, shown as a Mollweide projection: experimental data, above; theoretical prediction, below. Red indicates outward flux, blue, inward. The agreement in degree l, order (azimuthal wavenumber) m, frequency ω/Ω_o, and induction pattern between experimental data and theoretical predictions indicates the presence of inertial modes.

Over-reflection

What mechanism forces these inertial modes? A shear layer between two fluid regions can reflect waves, and Ribner (1957) showed that if the Mach number between the two regions M>2, over-reflection can amplify the waves. In our experiment,

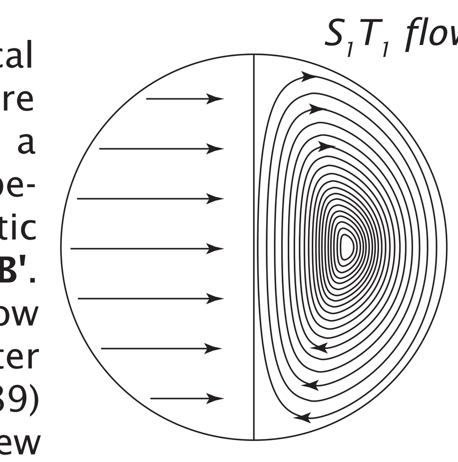
$$M = \frac{m\Omega_o}{\omega}(\chi - 1)$$

Setting M=2 and m=(1,2,3,4) yields the lines plotted on the spectrogram, above. Having these lines bound the regions where inertial waves are present, we conclude that over-reflection is the likely forcing mechanism.



Ongoing work

In considering the dynamo effect in spherical boundaries, Bullard and Gellman (1954) were able to produce on mathematical grounds a set of selection rules for the relationships between a flow field $\mathbf{u}$, an imposed magnetic field $\mathbf{B}_0$, and the induced magnetic field $\mathbf{B}'$. The authors went on to suggest a simple flow as likely candidate for dynamo action. Later numerical work (Dudley and James, 1989) predicted the onset of dynamo action in a few simple flows, including the S_2T_1 flow and the S_1T_1 flow, shown at right (toroidal flow shown on left half; cross section of poloidal flow shown on right half). Here the notation S_1T_1 indicates a superposition of vector spherical harmonics, $\varepsilon s_1^0+t_1^0$, where ε is a dimensionless parameter and

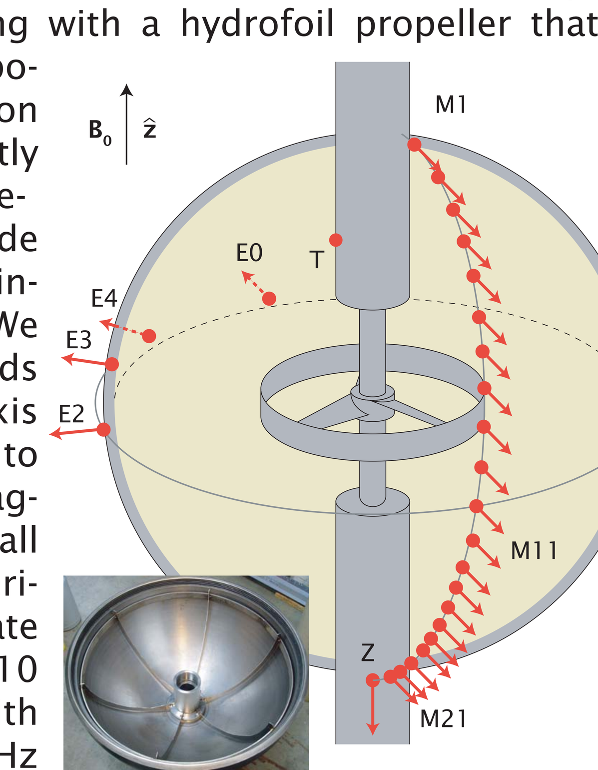


$$\mathbf{s_1^0} = \nabla\times\nabla\times\left(Y_1^0(\theta,\phi)\mathbf{r}\sin\pi r\right)$$
$$\mathbf{t_1^0} = \nabla\times\left(Y_1^0(\theta,\phi)\mathbf{r}\sin\pi r\right)$$

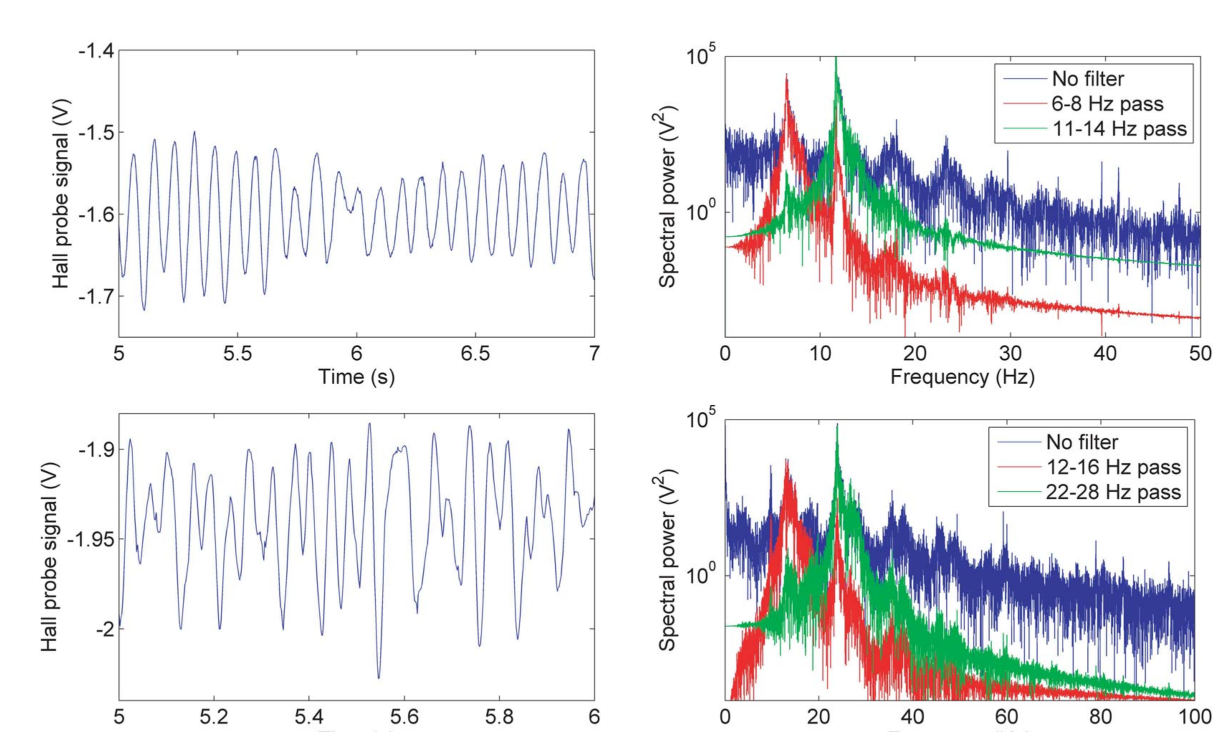
Spherical coordinates (r,θ,ϕ) are used, and $Y_1^0(\theta,\phi)$ is the scalar spherical harmonic with (l,m)=(1,0).

Experimental setup

We study S_1T_1 flows using the apparatus shown here, composed of the same 60 cm outer sphere described above, which rotates to approximate toroidal T_1 flow, along with a hydrofoil propeller that pumps axially to approximate poloidal S_1 flow. Because both rotation rates are adjustable, we can directly control the parameter ε. Six pole-to-pole baffles (see photo) protrude about 2.5 cm into the bulk to increase coupling to the sphere. We apply external, DC magnetic fields both parallel to the rotation axis (up to 400 G) and perpendicular to it (up to 50 G) and observe the magnetic response with a set of 26 Hall probes (see diagram). Initial experiments have explored rotation rate combinations -5 Hz < $\Omega_o/2\pi$ < 10 Hz and -35 Hz < $\Omega_i/2\pi$ < 35 Hz, with $|\Omega_o/2\pi|$ < 0.5 Hz and $|\Omega_i/2\pi|$ < 5 Hz being inaccessible because of motor limitations.



Results

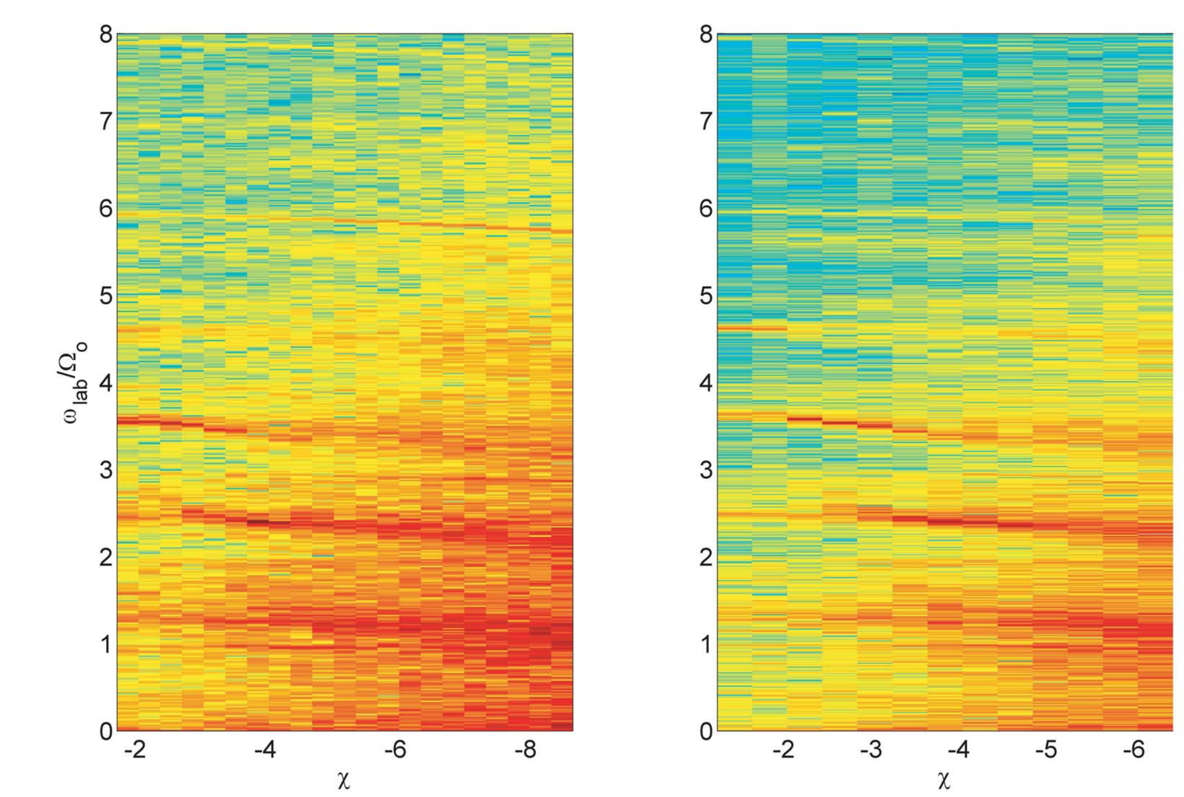


Two time series from a single Hall probe, along with their power spectra. Experimental parameters are $(\Omega_i,\Omega_o)/2\pi=(-13,5)$ Hz, upper; $(\Omega_i,\Omega_o)/2\pi=(-18,10)$ Hz, lower. Filtered data also shown; see below.

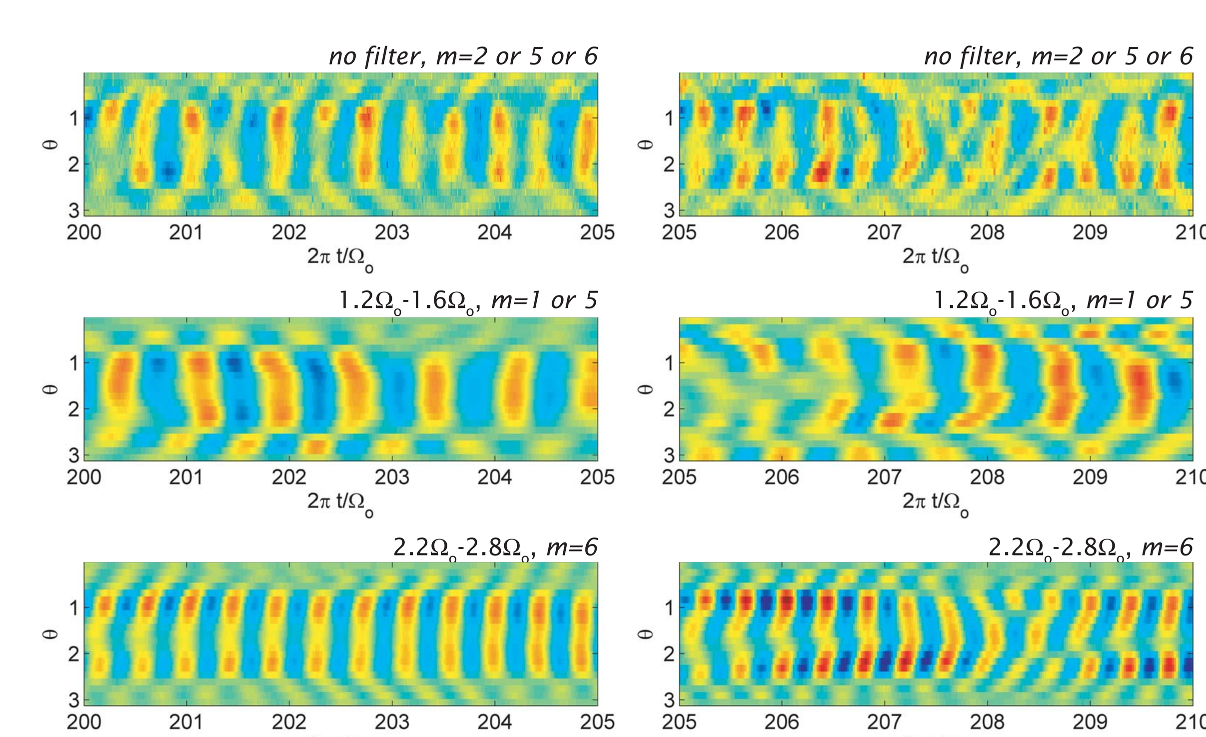
A broad range of behaviors have been observed; here we focus on a particular subset with common features in spatial structure and frequency. Two example time series from a single Hall probe are shown above, along with their power spectra. Note the peaks near $1.3\Omega_o$ and $2.4\Omega_o$, which are present in many regions of parameter space (see table, spectrograms below). At those parameter combinations, the induction at the surface of the sphere also has a consistent shape (see plots below). However, the patterns seem to have no well-defined azimuthal wavenumber, m. We have attempted to simplify the patterns by bandpass filtering ($1.2\Omega_o$ to $1.6\Omega_o$, in red above, and $2.2\Omega_o$ to $2.8\Omega_o$, in green above), however the azimuthal wavenumbers remain inconsistent and/or poorly defined. The detailed structure of this induction pattern—along with the physics that governs its excitation and dynamics—is the subject of ongoing study.

Ω_o (Hz)	Ω_o (Hz)	Leading m
-5 to -24	10	2
-5 to -25	5	2
-5 to -33	2	1
-5 to -26	3	3
5 to 26	-3	5
6 to 18	-5	5

Ranges of parameter space where crescent behavior appears.



Spectrograms of propeller ramps. $\Omega_o/2\pi$=-3 Hz, left, -5 Hz, right. Axes are as in above spectrogram.



Induction B_s/B_0 at the surface over five revolutions of the outer sphere. Red indicates outward flux, blue, inward. $\Omega_o/2\pi$=5 Hz, left, 10 Hz, right. Filter passband and strongest azimuthal wavenumber are as indicated.

References

Bullard, E. and Gellman, H., Homogeneous dynamos and terrestrial magnetism. *Philos. T. R. Soc. A*, 1954, **247**, 213-278.

Dudley, M. L. and James, R. W., Time-dependent kindmatic dynamos with stationary flows. *Philos. T. R. Soc. A*, 1989, **425**, 407-429.

Kelley, D. H. and Triana, S. A., and Zimmerman, D. S., and Tilgner, A., and Lathrop, D. P., Inertial waves driven by differential rotation in a planetary geometry. *Geophys. Astro. Fluid.*, currently in press.

Ribner, H. S., Reflection, transmission, and amplification of sound by a moving medium. *J. Acoust. Soc. Am.*, 1957, **29**, 435–441.

More information: http://complex.umd.edu · dhk@umd.edu